Surds

What is a surd ?

A surd is an irrational number in root form.

Irrational number:

A number that cannot be expressed as a simple fraction i.e. $\frac{a}{b}$

You can think of surds as being square roots of numbers that do not have a whole number as the root.

Examples of surds: $\sqrt{2},\ \sqrt{3},\ \sqrt{5},\ \sqrt{6},\ \sqrt{7},\ \sqrt{8},\ \sqrt{10}\ \ etc.$

the following are not surds:

$$\sqrt{4},\ \sqrt{9},\ \sqrt{16},\ \sqrt{25},\ \sqrt{36},\ \sqrt{49}\ \ etc.$$

as these do have a whole number as their roots.

Surds crop up in many situations in mathematics and we will learn some basic rules to deal with them.

Rules of surds:

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

these are true in both directions.

i.e.

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Applications:

We usually want to simplify expressions involving surds.

Example:

Simplify $\sqrt{75}$ [Hint: look for factors of 75 that are perfect squares; in this case 25]

Solution:

$$\sqrt{75} = \sqrt{25\times 3} = \sqrt{25}\times\sqrt{3} = 5\sqrt{3}$$

Example: Simplify $\sqrt{32}$

Solution: $\sqrt{32} = \sqrt{16\times 2} = \sqrt{16}\times\sqrt{2} = 4\sqrt{2}$

Example: Simplify $\dfrac{\sqrt{72}}{\sqrt{3}}$

Solution: $\dfrac{\sqrt{72}}{\sqrt{3}} = \sqrt{\dfrac{72}{3}} = \sqrt{24} = \sqrt{4\times 6} = \sqrt{4}\sqrt{6} = 2\sqrt{6}$

Adding and subtracting surds:

Just as in algebra – like terms can be added and subtracted

e.g. Simplify: $5\sqrt{3}-2\sqrt{3} \rightarrow 5\sqrt{3}-2\sqrt{3} \rightarrow 3\sqrt{3}$

e.g. Simplify: $4\sqrt{2}-3\sqrt{2} \rightarrow 4\sqrt{2}-3\sqrt{2} \rightarrow 1\sqrt{2} \rightarrow \sqrt{2}$

e.g. Simplify: $5\sqrt{7}+3\sqrt{7} \rightarrow 5\sqrt{7}+3\sqrt{7} \rightarrow 8\sqrt{7}$

e.g. Simplify: $3\sqrt{2}+\sqrt{2} \rightarrow 3\sqrt{2}+\sqrt{2} \rightarrow 4\sqrt{2}$

Past Paper Questions:

1. Express $\sqrt{50}$ as a surd in its simplest form.

Ans: $\sqrt{25\times 2} \to 5\sqrt{2}$

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2. Simplify $\sqrt{48} - 3\sqrt{3}$

Ans: $\sqrt{16\times 3} - 3\sqrt{3} \to 4\sqrt{3} - 3\sqrt{3} \to \sqrt{3}$

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3. Express $\sqrt{32} - \sqrt{2}$ as a surd in its simplest form.

Ans: $\sqrt{32} - \sqrt{2} \to \sqrt{16\times 2} - \sqrt{2} \to 4\sqrt{2} - \sqrt{2} \to 3\sqrt{2}$

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5. Express $\sqrt{72} - \sqrt{2} + \sqrt{50}$ as a surd in its simplest form

Ans: $\sqrt{72} - \sqrt{2} + \sqrt{50} \to \sqrt{36\times 2} - \sqrt{2} + \sqrt{25\times 2} \to 6\sqrt{2} - \sqrt{2} + 5\sqrt{2} \to 10\sqrt{2}$

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6. Express $\sqrt{32} + \sqrt{8}$ as a surd in its simplest form.

Ans: $\sqrt{32} + \sqrt{8} \to \sqrt{16\times 2} + \sqrt{4\times 2} \to 4\sqrt{2} + 2\sqrt{2} \to 6\sqrt{2}$

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7. Multiply out the brackets $\sqrt{2}\left(\sqrt{6} - \sqrt{2}\right)$

Express your answer as a **surd** in its simplest form.

Ans: $\sqrt{2}\times\sqrt{6} - \sqrt{2}\times\sqrt{2} \to \sqrt{12} - \sqrt{4} \to \sqrt{4\times 3} - 2 \to 2\sqrt{3} - 2$

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8. $f(x) = 3\sqrt{x}$

Find the exact value of $f(12)$, giving your answer as a **surd, in its simplest form**.

Ans: $f(12) = 3\sqrt{12} \to 3\sqrt{4\times 3} \to 3\times\sqrt{4}\times\sqrt{3} \to 6\sqrt{3}$

Rationalising the denominator:

When we have a surd in the denominator, it is better to simplify it further.

When we remove the surd from the denominator, we are "rationalising the denominator".

e.g. Rationalise the denominator.

$\frac{5}{\sqrt{2}}$ All we do is multiply top and bottom by the surd in the denominator.

$\rightarrow \frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ which is the same as multiplying by 1

$$\rightarrow \frac{5\sqrt{2}}{\sqrt{2}\times\sqrt{2}} \rightarrow \frac{5\sqrt{2}}{\sqrt{2\times 2}} \rightarrow \frac{5\sqrt{2}}{\sqrt{4}} \rightarrow \frac{5\sqrt{2}}{2}$$

Clearly this will always work since: $\sqrt{a}\times\sqrt{a} \rightarrow \sqrt{a\times a} \rightarrow \sqrt{a^2} \rightarrow a$

Examples:

Rationalise the denominators:

1. $\frac{7}{\sqrt{3}}$ $\rightarrow \frac{7}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} \rightarrow \frac{7\sqrt{3}}{3}$

2. $\frac{1}{\sqrt{6}}$ $\rightarrow \frac{1}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}} \rightarrow \frac{\sqrt{6}}{6}$

Past Paper Questions:

1. Express $\frac{3}{\sqrt{5}}$ as a fraction with a rational denominator.

2. Simplify $\frac{\sqrt{3}}{\sqrt{24}}$ Express your answer as a fraction with a rational denominator

3. $f(x) = \frac{3}{\sqrt{x}}$ Find the **exact** value of *f*(2)

   Give your answer **as a fraction** with a rational denominator.

Solutions:

1. $\frac{3}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} \rightarrow \frac{3\sqrt{5}}{5}$

2. $\sqrt{\frac{3}{24}} \rightarrow \sqrt{\frac{1}{8}} \rightarrow \frac{1}{\sqrt{4\times 2}} \rightarrow \frac{1}{2\sqrt{2}} \rightarrow \frac{1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{\sqrt{2}}{4}$

3. $f(2) = \frac{3}{\sqrt{2}} \rightarrow \frac{3\sqrt{2}}{\sqrt{2}\sqrt{2}} \rightarrow \frac{3\sqrt{2}}{2}$