Proportion and Variation

Direct Proportion:

If y is directly proportional to x,

this means that y and x behave the same way;

double one – double the other;

we say y varies directly as x;

and we can write:

$$y \propto x \quad \text{i.e.} \quad y = kx \quad \text{where } k \text{ is a constant.}$$

Direct proportion can be checked by:

checking the gradient ratios are all the same;

drawing a graph – straight line through the origin.

Inverse Proportion:

If y is inversely proportional to x,

this means that y and x behave in opposite ways;

double one – halve the other;

we say y varies inversely as x;

and we can write:

$$y \propto \frac{1}{x} \quad \text{i.e.} \quad y = \frac{k}{x} \quad \text{or} \quad xy = k \quad \text{where } k \text{ is a constant.}$$

Inverse proportion can be checked by:

checking the product xy are all the same (i.e. = k)

drawing a graph – hyperbola.

Joint Variation

There are occasions where more than one variable is involved, each one varying in a different manner.

e.g. T varies directly as x and inversely as y.

$T \propto x \qquad T \propto \frac{1}{y}$ these can be combined to give: $T = k\frac{x}{y}$

More examples of Joint Variation

1. V varies directly as s and inversely as t^2 $V = k\frac{s}{t^2}$ or $V = \frac{ks}{t^2}$

2. T varies directly as x and inversely as the square root of y $T = k\frac{x}{\sqrt{y}}$ or $T = \frac{kx}{\sqrt{y}}$

3. P varies directly as T^2 and inversely as V^3 $P = \frac{kT^2}{V^3}$

Finding the constant

In order to find the constant k, we need to have some values to put into the formula.

Example:

M varies directly as t and inversely as v^2.
M = 60 when t = 20 and v = 5

a) Find a formula for M in terms of t and v.

b) Calculate M when t = 9 and v = 9

Solution:

Find the formula: $M = \frac{kt}{v^2}$

Find the value of k $60 = \frac{k \times 20}{5^2}$ (substituting the given values)

$\frac{60 \times 25}{20} = k$ hence k = 75

Formula is: $M = \frac{75t}{v^2}$

Now use the formula: $M = \frac{75 \times 9}{9^2}$ (substitute t = 9 and v = 9)

$M = 8\frac{1}{3}$

Past Paper Questions

1. A weight on the end of a string is spun in a circle on a smooth table.

   The tension, *T*, in the string varies directly as the square of the speed, *v*, and inversely as the radius, *r*, of the circle.

   a) Write down a formula for T in terms of *v* and r.

   b) The speed of the weight is multiplied by 3 and the radius of the string is halved.

   What happens to the tension in the string.

Solution:

1. a) $T = \frac{kv^2}{r}$

   b) If v is multiplied by 3 then v^2 in the formula causes T to be multiplied by 9

   If r is halved then T is doubled. So overall effect is to multiply T by 18

2. The electrical resistance, *R*, of copper wire varies directly as its length, *L* metres, and inversely as the square of its diameter, *d* millimetres .

   Two lengths of copper wire, A and B, have the same resistance.

   Wire A has a diameter of 2 millimetres and a length of 3 metres.

   Wire B has a diameter of 3 millimetres

   What is the length of wire B.

Solution:

2. a) $R = \frac{kL}{d^2}$

   b) Wire A: $R = \frac{3k}{2^2}$ Wire B: $R = \frac{kL}{3^2}$

   Since resistance is same for both wires $\frac{kL}{3^2} = \frac{3k}{2^2}$ so, $\frac{\cancel{k}L}{3^2} = \frac{3\cancel{k}}{2^2} \rightarrow L = \frac{3 \times 3^2}{2^2} = \frac{27}{4} = 6.75\ m$

   Length of wire B is 6.75 metres.

3. A frictional force is necessary for a car to round a bend.

The frictional force, *F* kilonewtons, varies directly as the square of the car's speed, *V* metres per second, and inversely as the radius of the bend, *R* metres.

a) Write down a relationship between F, V and R.

A frictional force of 20 kilonewtons is necessary for a car, travelling at a given speed to round a bend.

b) Find the frictional force necessary for the same car, travelling at **twice** the given speed, to round the same bend.

Solution:

3. a) $F = \dfrac{kV^2}{R}$

b) If V is multiplied by 2,
then V^2 will cause F to be multiplied by 4

So the frictional force will be 80 kilonewtons

4. The time, *T* minutes, taken for a stadium to empty varies directly as the number of spectators, *S*, and inversely as the number of open Exits, *E*.

a) Write down a relationship connecting *T*, *S* and *E*.

It takes 12 minutes for a stadium to empty when there are 20 000 spectators and 20 open exits.

b) How long does it take the stadium to empty when there are 36 000 spectators and 24 open exits ?

Solution:

4. a) $T = \dfrac{kS}{E}$

b) T = 12 , when S = 20 000 and E = 20 $\quad 12 = \dfrac{20000k}{20} \rightarrow 12 = 1000k \rightarrow k = \dfrac{12}{1000}$

c) $T = \dfrac{12}{1000} \times \dfrac{36000}{24} \rightarrow \dfrac{36}{2} \rightarrow 18 \text{ minutes}$

5. The number of litres of petrol, *L*, used by a car on a journey varies directly

as the distance, *D* kilometres, travelled, and as the square root of the average speed, *S* kilometres per hour.

a) Write down a relationship connecting *L*, *D* and *S*.

The car uses 30 litres of petrol for a journey of 550 kilometres when it travels at an average speed of 81 kilometres per hour.

b) How many litres of petrol does the car use for a journey of 693 kilometres travelling at an average speed of 100 kilometres per hour.

Solution:

5. a) $L = kD\sqrt{S}$

b) L = 30, when D = 550 and S = 81

$30 = k \times 550\sqrt{81} \rightarrow k = \frac{30}{550 \times 9} = \frac{1}{165}$

c) $L = \frac{1}{165} \times 693 \times \sqrt{100} = 42$ litres

6. The surface area of a planet, *A* square kilometers, varies directly as the square of the diameter, *D* kilometres of the planet.

The surface area of the Moon is 3.8×10^7 square kilometres.

Calculate the surface area of a planet with diameter double the diameter of the Moon. **Give your answer in scientific notation**. 3 KU

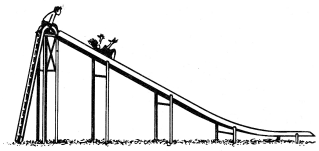
Solution:

6. $A = kD^2$

For Moon: surface area = 3.8×10^7 If diameter is multiplied by 2 then A will be $\times$ 4

Hence surface area of planet = $4 \times 3.8 \times 10^7 = 1.52 \times 10^8$ km^2

7. The time, T seconds, taken by a child to slide down a chute varies directly as the length, L metres, of the chute and inversely as the square root of the height, H metres, of the chute above the ground.



It takes 10 seconds to slide down a chute which is 3.75 metres long and 2.25 metres high.

a) Find a formula connecting T, L and H.

b) How long does it take to slide down a chute which is 5 metres long and 2.56 metres high?

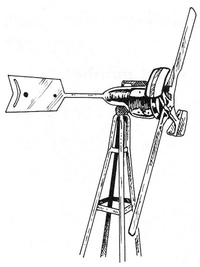
Solution:

8. a) $T = \frac{kL}{\sqrt{H}}$

b) T = 10, when L = 3.75 and H = 2.25

$$10 = \frac{k \times 3.75}{\sqrt{2.25}} \rightarrow k = \frac{10 \times \sqrt{2.25}}{3.75} = 4 \qquad T = \frac{4 \times 5}{\sqrt{2.56}} \rightarrow 12.5 \text{ seconds}$$

9. The power, P watts, produced by a windmill varies directly as the cube of the wind velocity, V metres per second.



At 4 pm on a given day, the wind velocity was 4 metres per second and the windmill was producing 75 watts of electrical power.

By 10 pm the wind velocity had doubled.

How many watts of electrical power were now being produced?

Solution:

9. $P = kV^3$

P = 75, when V = 4 $\qquad 75 = k \times 4^3 \rightarrow k = \frac{75}{64}$

When wind speed doubled, V = 8

$$P = \frac{75}{64} \times 8^3 \rightarrow \frac{75 \times 8 \times 64}{64} = 600 \text{ watts}$$

Alternative way:

$P = kV^3$ if V is doubled,

then P will be multiplied by 2^3 or 8

Hence Power will be 75 x 8 = 600 watts.