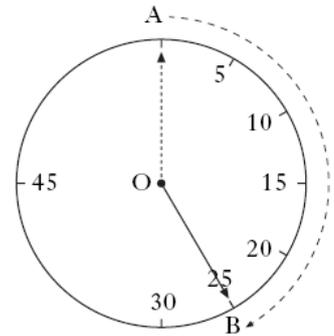**Problem Solving Questions 3**

1. Contestants in a quiz have 25 seconds to answer a question.

    This time is indicated on the clock.

   The tip of the clock hand moves through the arc AB as shown.

    (*a*)   Calculate the size of angle AOB.   **1 KU**

    (*b*)   The length of arc AB is 120 centimetres.

            Calculate the length of the clock hand.   **4 RE**

2. A cone is formed from a paper circle with a sector removed as shown.
    The radius of the paper circle is 30 cm.
    Angle AOB is 100°.
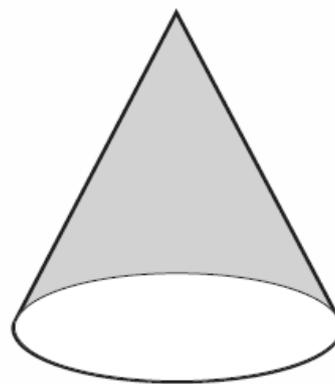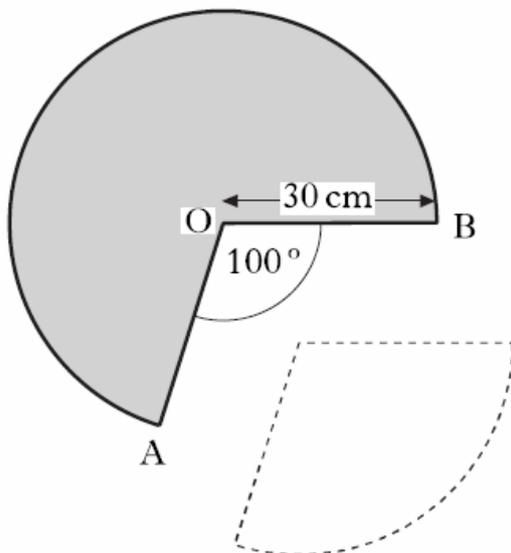
    a)   Calculate the area of paper used to make the cone.   **3 KU**

    b)   Calculate the circumference of the base of the cone.   **3 RE**

3.	In triangle ABC

BC = 8 centimetres

AC = 6 centimetres

PQ is parallel to BC
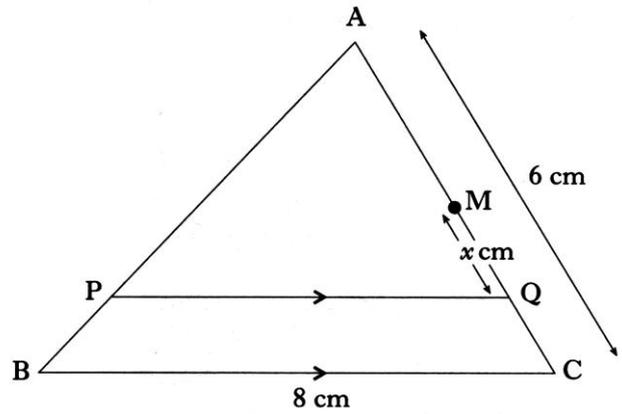
M is the mid-point of AC
Q lies on AC, x centimetres from M,
as shown on the diagram.

	(a)	Write down an expression for the length of AQ.	**1 RE**

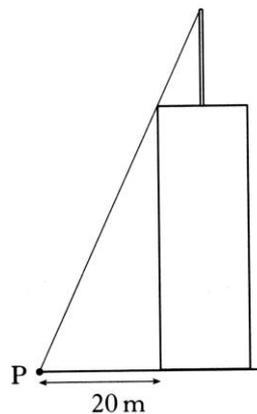	(b)	Show that $PQ = \left(4 + \dfrac{4}{3}x\right)$	**3 RE**

4.	A vertical flagpole 12 metres high stands
	at the centre of the roof of a tower.

	The tower is cuboid shaped with a square
	base of side 10 metres.

	At a point P on the ground, 20 metres
	from the base of the tower, the top of the
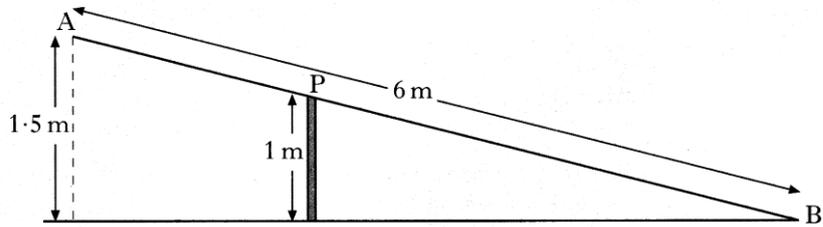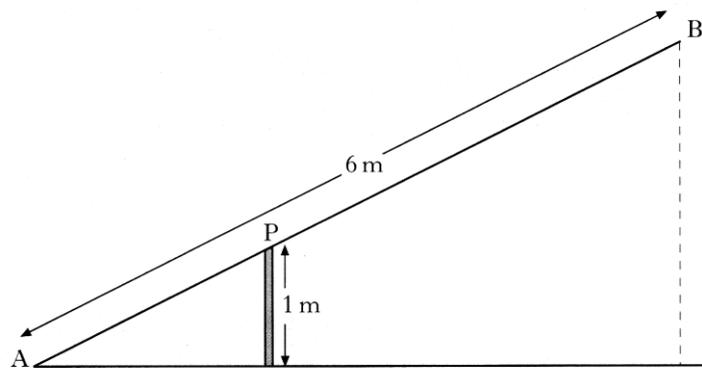	flagpole is just visible, as shown.

	Caclulate the height of the tower.	**4 RE**

5. A metal beam, AB, is 6 metres long.
    It is hinged at the top, P, of a vertical post 1 metre high.
    When B touches the ground, A is 1.5 metres above the ground, as shown
    In Figure 1.

    Figure 1

    When A comes down to the ground, B rises, as shown in Figure 2.

    By calculating the length of AP, or otherwise, find the height of B above
    the ground.

    Do not use a scale drawing.   **5 RE**
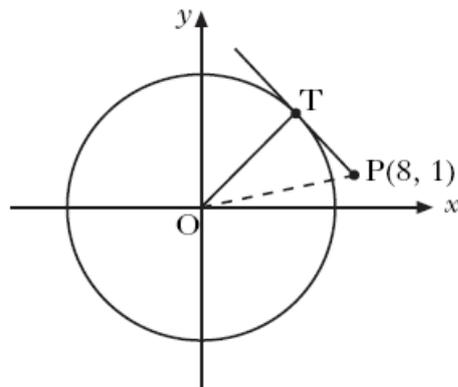
6. A circle, centre the origin, is shown.
    P is the point (8, 1).

    (a)   Calculate the length of OP.   **2 RE**

    The diagram also shows a tangent from P
    which touches the circle at T.
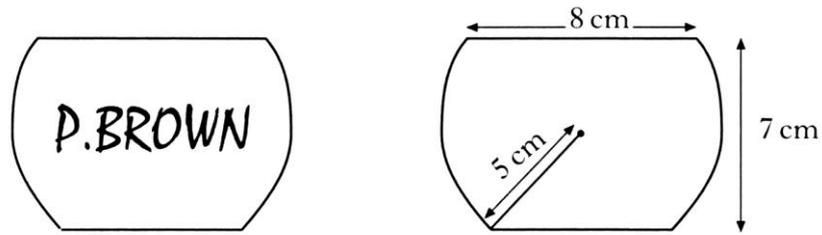   The radius of the circle is 5 units.

    (b)   Calculate the length of PT.   **2 RE**

7.   A badge is made from a circle of radius 5 centimetres.

Segments are taken off the top and the bottom of the circle as shown.

The straight edges are parallel.

The badge measures 7 centimetres from the top to the bottom.

The top is 8 centimetres wide.

Calculate the width of the base.   **5 RE**
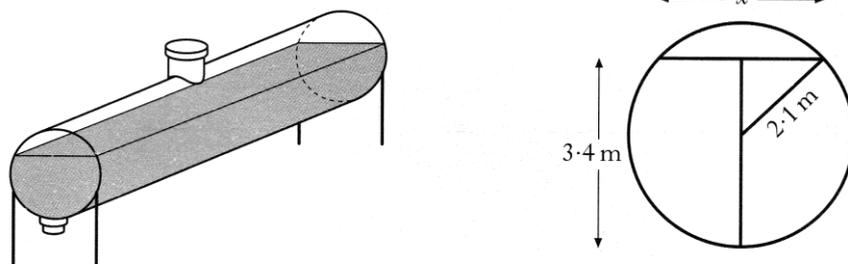

8.   The curved part of a doorway is an arc of a circle with radius
500 millimetres and centre C.

The height of the doorway to the top of the arc is 2000 millimetres.

The vertical edge of the doorway is 1800 millimetres.
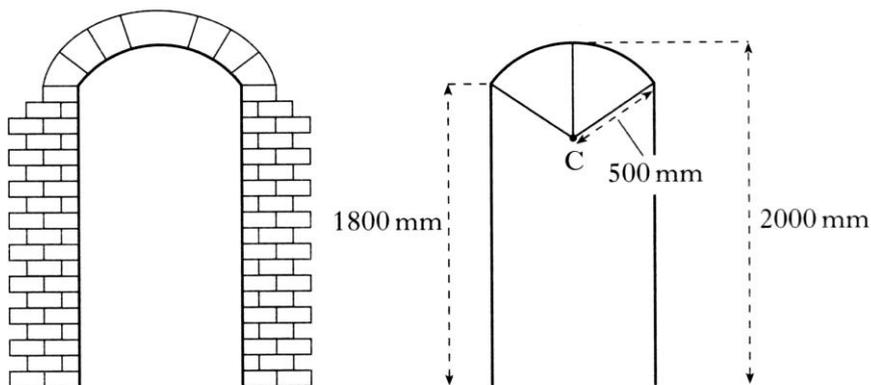
Calculate the width of the doorway.   **5 RE**

9.   An oil tank has a circular cross section of radius 2.1 metres.
It is filled to a depth of 3.4 metres.

(a)   Calculate x, the width in metres of the oil surface.   **3 KU**

(b)   What other depth of oil would give the same surface width.   **1 RE**

10. A sheep shelter is part of a cylinder
as shown in Figure 1.
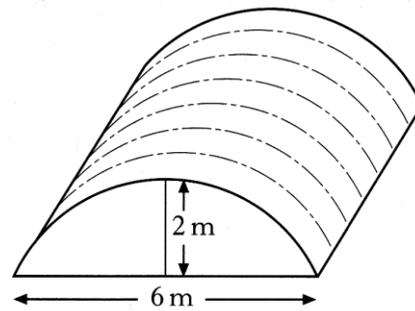
It is 6 metres wide and 2 metres high.

2 m

6 m

Figure 1

The cross-section of the shelter is a
segment of a circle with centre O,
as shown in Figure 2.

OB is the radius of the circle.

Calculate the length of OB.

6 m

2 m

A   B

O

Figure 2

4 RE

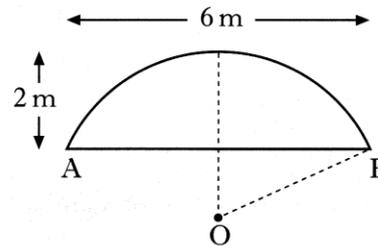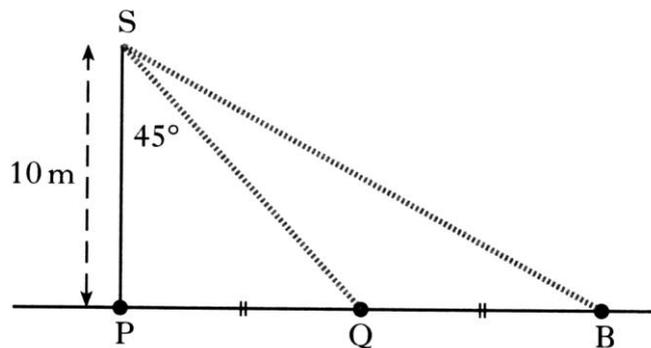11. The diagram below shows a spotlight at point S, mounted 10 metres directly
above a point P at the front edge of a stage.

The spotlight swings 45° from the vertical to illuminate another point Q,
also at the front edge of the stage.

S

45°

10 m

P   Q   B

Through how many more degrees must the spotlight swing to illuminate
a point B, where Q is the mid–point of PB ?

5 RE

12. A TV signal is sent from a transmitter T, via a satellite S, to a village V, as shown in the diagram. The village is 500 kilometres from the transmitter.
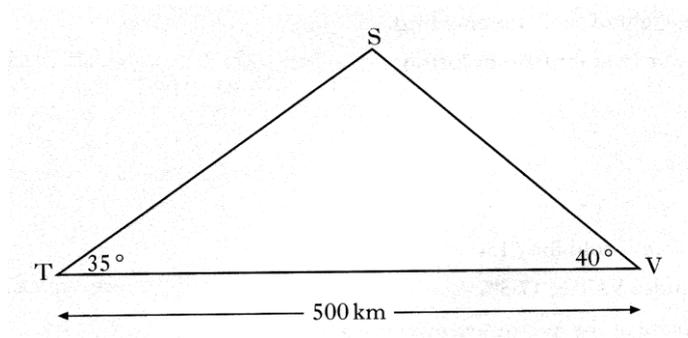
The signal is sent out at an angle of 35° and is received in the village at an angle of 40°.

Calculate the height of the satellite above the ground. **5 RE**

13. In triangle PQR

QR = 6 centimetres

Angle PQR = 30°

Area of triangle PQR = 15 square centimetres.
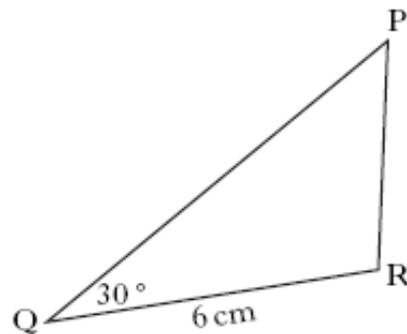
Calculate the length of PQ.

3 RE

14. Emma goes on the "Big Eye".
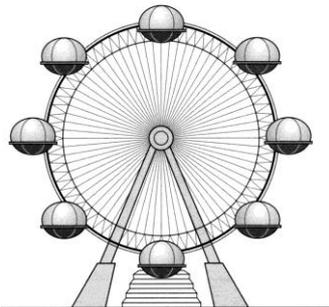
Her height, h metres, above the ground is giv

$$h = -31\cos t° + 33$$

where t is the number of seconds after the start.

(a) Calculate Emma's height above the ground 20 seconds after the start. **2 KU**

(b) When will Emma first reach a height of 60 metres above the ground ? **3 RE**

(c) When will she next be at a height of 60 metres above the ground. **1 RE**

15. Given $f(x) = 4\sqrt{x} + \sqrt{2}$

    (a) Find the value of $f(72)$ as a surd in its simplest form.   **3 KU**

    (b) Find the value of t, given that $f(t) = 3\sqrt{2}$.   **3 RE**

16. The sum S_n of the first n terms of a sequence, is given by the formula

    $$S_n = 3^n - 1$$

    (a) Find the sum of the first 2 terms.   **1 RE**
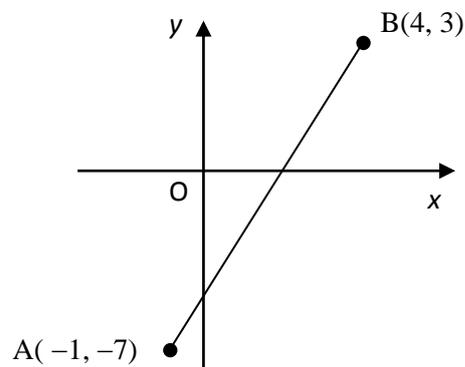
    (b) When $S_n = 80$, calculate the value of n.   **2 RE**

17. In the diagram, A is the point (−1, 7)
    and B is the point (4, 3).

    (a) Find the gradient of the line AB.   **1 KU**

    (b) AB cuts the *y*-axis at the point (0, −5).
        Write down the equation of the line AB   **1 KU**

    (c) The point (3*k*, *k*) lies on AB
        Find the value of k.   **2 RE**

18. The monthly bill for a mobile phone is made up of a fixed rental plus call
    charges. Call charges vary as the time used.

    The relationship between the monthly bill, y (pounds), and the time used,
    x (minutes) is represented in the graph below.
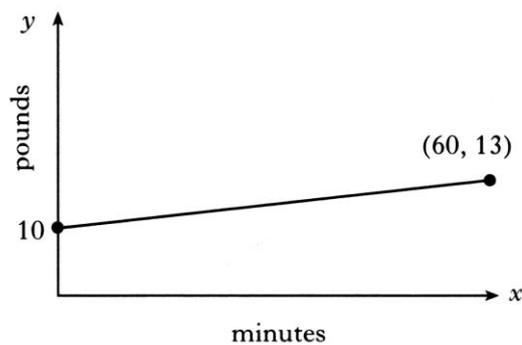
    (a) Write down the fixed rental.   **1 RE**

    (b) Find the call charge per minute.   **3 RE**

19. The profit made by a publishing company of a magazine is calculated by the formula

$$y = 4x(140 - x),$$

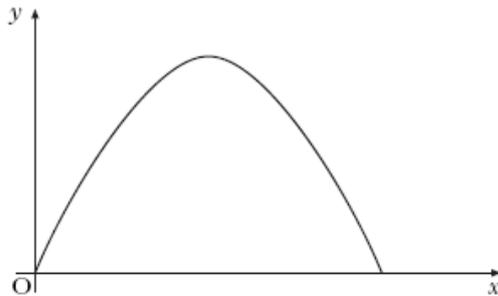where y is the profit (in pounds) and x is the selling price (in pence) of the magazine.
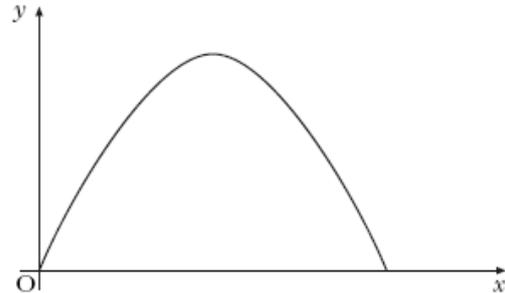The graph below represents the profit y against the selling price x.

Find the maximum profit the company can make from the sale of the magazine.   **4 RE**

20. The diagram shows part of the graph of a quadratic function, with equation of the form

$$y = k(x - a)(x - b)$$

The graph cuts the y-axis at (0, −6) and the x-axis at (−1, 0) and (3, 0)

(a) Write down the values of a and b.   **2 KU**

(b) Calculate the value of k.   **2 KU**

(c) Find the coordinates of the minimum turning point of the function   **2 RE**

21. A rectangular wall vent is 30 centimetres long and 20 centimetres wide.

It is to be enlarged by increasing **both** the length and the width by x centimetres.

(a) Write down the length of the new vent.   **1 RE**

(b) Show that the Area, A, square centimetres, of the new vent is given by

$$A = x^2 + 50x + 600$$ **2 RE**

(c) The area of the new vent must be at least 40% more than the original area.

Find the minimum dimensions to the nearest centimetre, of the new vent.   **5 RE**

22. The number of diagonals, d, in a polygon with n sides is given by the formula:

$$d = \frac{n(n-3)}{2}$$

A polygon has 20 diagonals

How many sides does it have? **4 RE**

23. A number pattern is given below.

1st term: $2^2 - 0^2$
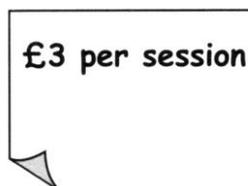
2nd term: $3^2 - 1^2$

3rd term: $4^2 - 2^2$

(a) Write down a similar expression for the 4th term. **1 RE**

(b) Hence or otherwise find the nth term in its simplest form. **3 RE**

24. (a) One session at the Leisure Centre costs £3.

£3 per session

Write down an algebraic expression for the cost of x sessions **1 RE**

(b) The Leisure Centre also offers a monthly card costing £20.
The first 6 sessions are then free, with each additional session costing £2.

Monthly card
£20

* first 6 sessions **free**
* each additional
  session £2

(i) Find the total cost of a monthly card and 15 sessions. **1 KU**

(ii) Write down an algebraic expression for the total cost of a monthly
card and x sessions where x is greater than 6. **2 RE**

(c) Find the minimum number of sessions required for the monthly card
to be the cheaper option.

Show all working. **3 RE**

25. A new fraction is obtained by adding x to the numerator and denominator of the fraction $\dfrac{17}{24}$.

This new fraction is equivalent to $\dfrac{2}{3}$.

Calculate the value of x.   **3 RE**

26. To hire a car costs £25 per day plus a mileage charge.

CAR HIRE
£25 per day
· **first 200** miles free
· each additional mile only 12p

    The first 200 miles are free with each additional mile charged at 12 pence.

    (a) Calculate the cost of hiring a car for 4 days when the mileage is 640 miles.   **1 KU**

    (b) A car is hired for d days and the mileage is m miles where $m > 200$.
    Write down a formula for the cost £C of hiring the car.   **3 RE**

27. The nth term, T_n of the sequence 1, 3, 6, 10 ….. is given by the formula:

$T_n = \dfrac{1}{2}n(n+1)$ 1st term $T_1 = \dfrac{1}{2}\times 1(1+1) = 1$

2nd term $T_2 = \dfrac{1}{2}\times 2(2+1) = 3$

3rd term $T_3 = \dfrac{1}{2}\times 3(3+1) = 6$

(a) Calculate the 20th term, T_{20}.   **1 KU**

(b) Show that $T_{n+1} = \dfrac{1}{2}\left(n^2 + 3n + 2\right)$ **2 RE**

(c) Show that $T_n + T_{n+1}$ is a square number   **2 RE**

28. A number pattern is shown below.

$$1^3 = \frac{1^2 \times 2^2}{4}$$

$$1^3 + 2^3 = \frac{2^2 \times 3^2}{4}$$

$$1^3 + 2^3 + 3^3 = \frac{3^2 \times 4^2}{4}$$

(a) Write down a similar expression for: $1^3 + 2^3 + 3^3 + 4^3 + 5^3$ **1 RE**

(b) Write down a similar expression for: $1^3 + 2^3 + 3^3 ...... + n^3$ **2 RE**

(c) Hence **evaluate**: $1^3 + 2^3 + 3^3 ...... + 9^3$ **2 RE**