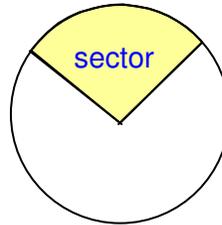


# Fractions of a circle

## Parts of the circle:

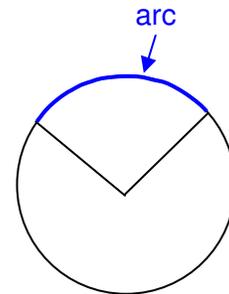
### Sector:

A sector of a circle, is the area formed between 2 radii of the circle and the circumference.



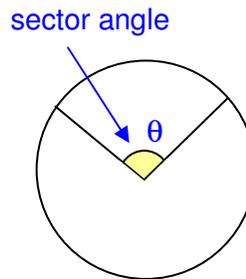
### Arc:

An arc of a circle, is the part of the circumference cut off by 2 radii of the circle.



### Sector angle:

The angle between the two radii, that define a sector or an arc.



The Greek letter 'theta'  $\theta$  is often used to label this angle.

## Two useful formulae.

We can calculate the **area of a sector**.

The **area of the sector** is the same fraction of the **area of the circle**, as the **sector angle** is of  $360^\circ$

i.e 
$$\text{Area of sector} = \frac{\text{sector angle}}{360^\circ} \times \pi r^2$$

We can calculate the **length of an arc** (arc length).

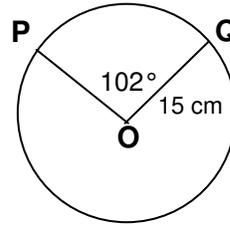
The **length of the arc** is the same fraction of the **circumference of the circle**, as the **sector angle** is of  $360^\circ$

i.e 
$$\text{Arc length} = \frac{\text{sector angle}}{360^\circ} \times \pi d$$

**Example:**

Find the area of the sector OPQ in the following diagram.

Where angle POQ =  $102^\circ$   
and the radius of the circle is 15 cm.



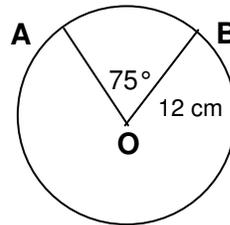
$$\text{Area of sector} = \frac{\text{sector angle}}{360^\circ} \times \pi r^2$$

$$\text{Area of sector} = \frac{102}{360} \times \pi \times 15^2 = 200.276\dots = 200.3 \text{ cm}^2.$$

**Example:**

Find the length of the arc AB in the following diagram.

Where angle AOB =  $75^\circ$   
and the radius of the circle is 12 cm.



$$\text{Arc length} = \frac{\text{sector angle}}{360^\circ} \times \pi d$$

Note we require the diameter. Diameter = 24 cm.

$$\text{Arc length} = \frac{75}{360} \times \pi \times 24 = 15.707\dots = 15.7 \text{ cm.}$$

**Examples to try:**

1. Find the area of the sector EOD,  
where the sector angle is  $125^\circ$   
and the radius of the circle is 22 cm.

[ Ans:  $527.96 \text{ cm}^2$  ]

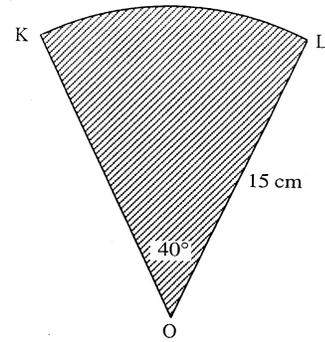
2. Find the length of the arc MN,  
where the sector angle MON is  $83^\circ$   
and the radius of the circle is 9 cm.

[ Ans: 13.04 cm ]

Past Paper Questions:

1. Sector KOL of a circle centre O and radius 15 centimetres is shown opposite.

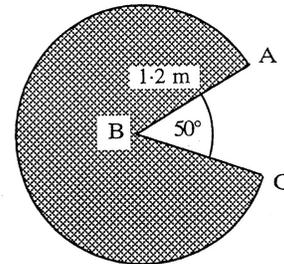
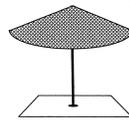
Calculate the area of this sector.



2. June is replacing the fabric on her garden parasol.

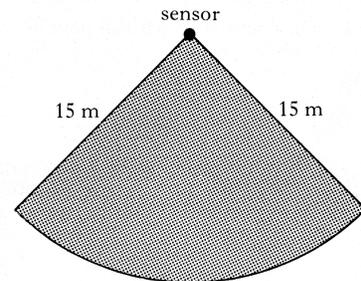
She uses a sector of a circle, with radius 1.2 metres.

Calculate the area of fabric needed to replace the old material.



3. A sensor in a security system covers a horizontal area in the shape of a sector of a circle of radius 15 m.

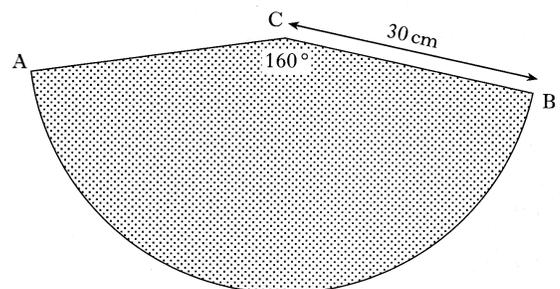
The area of the sector is 200 square metres.  
Find the length of the arc of the sector.



4. The diagram shows a sector of a circle, centre, C.

Angle ACB is 160°,  
and the radius of the circle is 30 cm.

Calculate the length of the arc AB.



**Solutions:**

1. Area of sector =  $\frac{40}{360} \times \pi \times 15^2 = 78.5 \text{ cm}^2$  (3 sf)

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2. Area of sector =  $\frac{50}{360} \times \pi \times 1.2^2 = 0.63 \text{ m}^2$  (2 sf)

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3. Let angle of sector =  $\theta$  So,

$$200 = \frac{\theta}{360} \times \pi \times 15^2$$

Re-arrange to get

$$\theta = \frac{200 \times 360}{\pi \times 15^2} = \frac{320}{\pi}$$

Length of arc:

$$\frac{\theta}{360} \times \pi \times 30 = \frac{320}{\pi} \times \frac{\pi \times 30}{360} = 26.7 \text{ m}$$

**Alternatively,**  $\frac{\text{arc length}}{\text{circumference}} = \frac{\text{area of sector}}{\text{area of circle}}$

So,  $\frac{\text{arc length}}{\pi \times 30} = \frac{200}{\pi \times 15 \times 15}$ , arc length =  $\frac{200 \times \pi \times 30}{\pi \times 15 \times 15}$

arc length = 26.7 m

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4. Arc length =  $\frac{160}{360} \times \pi \times 60 = 83.8 \text{ cm}$  (3 sf)

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