CREDIT 2005 – Paper II

1. $E = mc^2$

Find the value of E when

$m = 3.6\times10^{-2}$ and $c = 3\times10^{8}$.

$E = 3.6 \times 10^{-2} \times 3 \times 10^{8} \times 3 \times 10^{8}$

Using calculator:

$E = 3.24 \times 10^{15}$

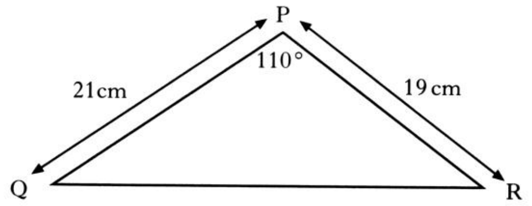
2.

	x	$x-\bar{x}$	$\left(x-\bar{x}\right)^2$
	77	–2.5	6.25
	91	11.5	132.25
	84	4.5	20.25
	71	–8.5	72.25
	79	–0.5	0.25
	75	–4.5	20.25
TOTAL	477		251.5

a) Mean = $\dfrac{\sum x}{n} = \dfrac{477}{6} = 79.5$

b) S.D. = $\sqrt{\dfrac{251.5}{5}} = \sqrt{50.3} = 7.092.... = 7.1$

3.



Use formulae: $Area = \frac{1}{2}ab\sin C$

$Area = 0.5\times21\times19\times\sin110°$

$Area = 187.468...$

Area of triangle PQR = 187 cm^2

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4. Use the quadratic formula

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$x^2+2x=9$ Rearrange to normal form.

$x^2+2x-9=0$

Identify a, b and c.

a = 1, b = 2, c = –9 and then substitute.

$x = \dfrac{-2\pm\sqrt{2^2-4(1)(-9)}}{2\times1}$ now simplify

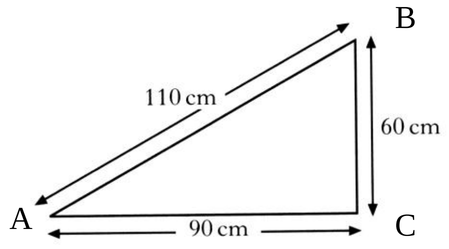
$x = \dfrac{-2\pm\sqrt{4+36}}{2} \Rightarrow x = \dfrac{-2\pm\sqrt{40}}{2}$

$\Rightarrow x = \dfrac{-2+\sqrt{40}}{2}$ *or* $x = \dfrac{-2-\sqrt{40}}{2}$

$\Rightarrow x = \dfrac{4.325}{2}$ *or* $x = \dfrac{-8.325}{2}$

Hence x = 2.2 or – 4.2 (1 d.p.)

5.



Use converse of Pythagoras:

Give the triangle some letters e.g. A, B, C

Longest side = 110 so, $AB^2 = 110^2 = 12{,}100$

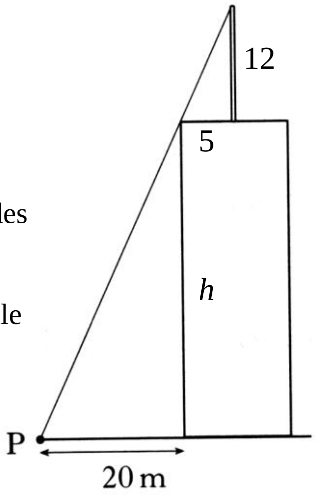
$AC^2 + CB^2 = 90^2 + 60^2 = 11{,}700$

Since, $AB^2 \neq AC^2 + CB^2$

The triangle is **NOT** right angled

(converse of Pythagoras)

6. Use similar triangles



Mark onto diagram what you know:

Tower is square, with sides 10 metres

Since flagpole is in middle base of top triangle is 5 m

Flagpole is 12 metres high.

Let height of building be *h* metres.

Noting that the two triangles are equiangular, therefore they are similar.

Method (i) – Using ratios

$\frac{h}{12} = \frac{20}{5}$ cross multiplying $h = \frac{20}{5} \times \frac{12}{1}$

So $h = 48$

Height of building = **48 metres**.

Method (ii) – Using scale factors

The base of the larger triangle is 4 times the size of the base of the smaller triangle at the top

So the scale factor of enlargement = 4

Hence the height of the building is 4 times the height of the flagpole.

So, height of building = 4×12 = **48 metres.**

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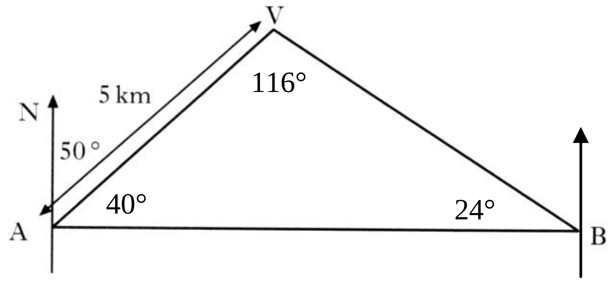
7. This is going to involve trigonometry

So we must ensure that everything we know

is marked onto the diagram.

7. *continued*



$\angle$ VAB = 90° – 50° = 40° *(B due East of A)*

V is on a bearing of 294° from B

So since A is due West, A is on a bearing of 270° from B.

Hence: $\angle$VBA = 294° – 270° = 24°

We now have two angles, so calculate angle AVB.

40 + 24 = 64; so $\angle$ VAB = 180 – 64 = 116°

Since we want to find AB and we have all three angles, use the Sine Rule:

$\frac{AB}{\sin 116°} = \frac{5}{\sin 24°}$ now cross multiply

$AB = \frac{5 \times \sin 116°}{\sin 24°}$ = 11.0488…

Length of AB = 11 km (nearest km)

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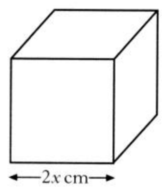
8. Volume of cube =

$2x \cdot 2x \cdot 2x \rightarrow 8x^3$



Area of one side of the cube =

$2x \cdot 2x \rightarrow 4x^2$

There are 6 faces so total surface area =

$6 \times 4x^2 \rightarrow 24x^2$

Since volume and surface area are numerically equal

$8x^3 = 24x^2$ now divide both sides by 8

$\rightarrow x^3 = 3x^2$

$\rightarrow x^3 - 3x^2 = 0$

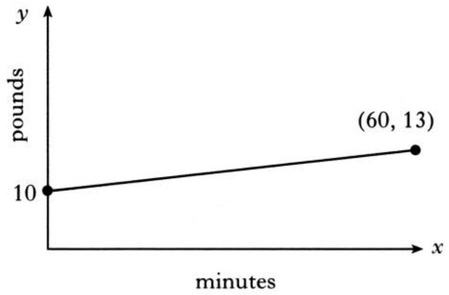
take out x^2 as common factor

$\rightarrow x^2(x-3) = 0$ so $x = 0$ or $\boldsymbol{x = 3}$

Length cannot be 0,

so side length of cube = **2*x*** = **6 cm**

9a).



Since for 0 minutes, cost is £10

Then the fixed rental is £10

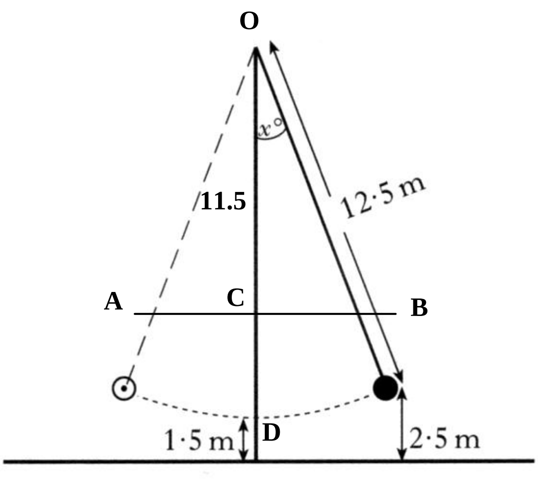
9b). After 60 minutes, the cost has gone up to £13

So 60 minutes of calls cost £3 or 300 pence

So, 1 minute costs 300 ÷ 60 = 5 pence

Call charge = 5p per minute.

10.



Label points on the diagram A, B, C, D, O as shown.
Join A and B with a horizontal line.

Since OD = 12.5 m (radius) and CD = 2.5 − 1.5 = 1 m

Then OC = 12.5 − 1 = 11.5 cm

Use SOH−CAH−TOA in triangle OCB

So: $\cos x = \dfrac{11.5}{12.5} \quad \Rightarrow \quad x = \cos^{-1}\left(\dfrac{11.5}{12.5}\right)$

and thus $x =$ 23.0739…

i.e. x is approximately 23°

10b). We now need to calculate the arc length
The angle through which it swings is:

$2 \times 23.0739\ldots$

$= 46.1°$

Arc length = $\dfrac{46.1}{360} \times \textit{Circumference}$

Circumference of circle = πd

Diameter of circle = 2 × 12.5 = 25 cm

$C = \pi \times 25$

Arc length = $\dfrac{46.1}{360} \times \pi \times 25$

Arc length = 10.0574…

Arc length = 10.1 cm (3 sig figs.)

11a). $\sqrt{3}\sin x - 1 = 0$

add 1 to both sides $\sqrt{3}\sin x = 1$

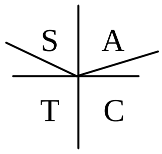
divide both sides by $\sqrt{3}$ $\rightarrow$ $\sin x = \dfrac{1}{\sqrt{3}}$

$x = \sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ (acute) x = 35.3°

Use ASTC

Sign is positive

So quadrants 1, 2



x = 35.3° and x = 180 – 35.3°

Solutions are: x = 35.3° and 144.7°

11b). $\sqrt{3}\sin 2x - 1 = 0$ on the same basis as above will give us:

$$2x = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$\Rightarrow$ $2x$ = 35.3° and $2x$ = 180 – 35.3 = 144.7°

so x = 17.7° and x = 72.4° (1 d.p.)

END OF QUESTION PAPER (*Rev. March 2007*)