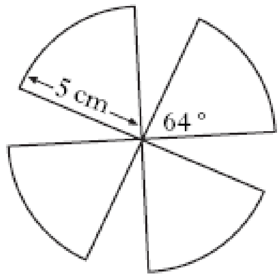
Banker KU Questions 4 - solutions

1. A fan has four identical plastic blades.



Each blade is a sector of a circle of radius 5 centimetres.

The angle at the centre of each sector is 64°

Calculate the total area of plastic required to make the blades. **3 KU**

Area of one blade $= \frac{64}{360} \times \pi \times 5^2 = 13.962....\ cm^2 \ \frac{10}{4} \rightarrow \frac{5}{2}$

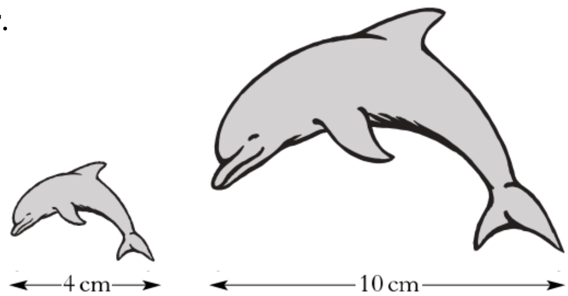
Total Area $= \frac{64}{360} \times \pi \times 5^2 \times 4 = 55.85....\ cm^2 = 55.9\, cm^2$

---

2. Two fridge magnets are mathematically similar.

One fridge magnet is 4 cm long

and the other is 10 cm long.



The area of the smaller magnet is 18 cm^2.

Calculate the area of the larger magnet. **3 KU**

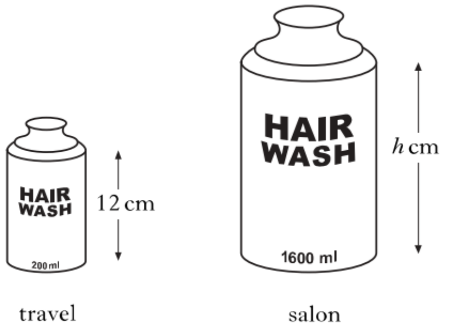
Length scale factor $= \frac{10}{4} \rightarrow \frac{5}{2}$

Area scale factor $=$ (length scale factor)^2 $= \frac{5}{2} \times \frac{5}{2} \rightarrow \frac{25}{4}$

Area of larger magnet $= 18 \times \frac{25}{4} \rightarrow \frac{\not{18}^{\,9}}{1} \times \frac{25}{\not{4}^{\,2}} \rightarrow \frac{225}{2} \rightarrow 112.5\ cm^2$

---

3. Shampoo is available in travel size and salon size bottles.
The bottles are mathematically similar.



The travel size contains 200 ml and is 12 cm in height.
The salon size contains 1600 ml.

Calculate the height of the salon size bottle. **3 KU**

Volume scale factor = $\dfrac{1600}{200} = 8$ Volume scale factor = (length scale factor)^3

Length scale factor = $\sqrt[3]{8} = 2$

Hence height of salon size bottle = $12 \times 2 = 24$ cm

---

4. (a) Solve algebraically the equation

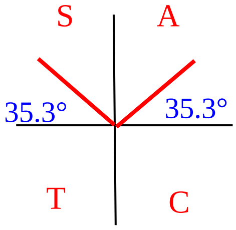
$$\sqrt{3}\sin x - 1 = 0 \qquad 0 \le x \le 360°$$ **3 KU**

$$\sqrt{3}\sin x = 1$$

$$\sin x = \frac{1}{\sqrt{3}}$$

$$acute\ x = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) = 35.3°$$

Use ASTC sine is positive, so quadrants 1 & 2.



Hence: $x = 35.3°$
Or $x = 180 - 35.3 = 144.7°$

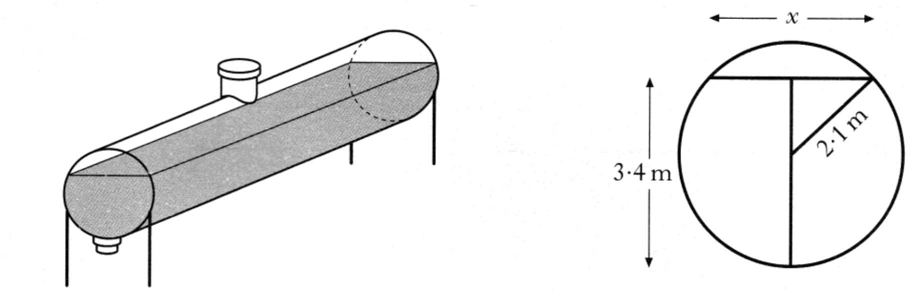
(b) Hence write down the solution of the equation

$$\sqrt{3}\sin x - 1 = 0 \qquad 0 \le x \le 90°$$ **1 RE**

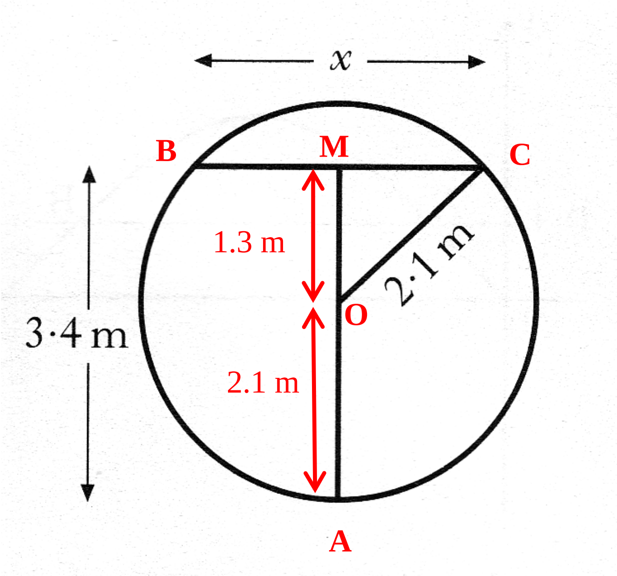
Solution required: $x = 35.3°$

---

5. An oil tank has a circular cross section of radius 2.1 metres.
It is filled to a depth of 3.4 metres.



(a) Calculate x, the width in metres of the oil surface. **3 KU**



Label the diagram as shown.

OA is a radius and so is 2.1 m

Hence OM = AM – 2.1 = 3.4 – 2.1 = 1.3 m

Using Pythagoras in triangle OMC.

$2.1^2 = 1.3^2 + MC^2$

Re-arranging,

$MC^2 = 2.1^2 - 1.3^2$

$MC^2 = 2.72$

MC = 1.649…

And so $x = 2 \times MC = 2 \times 1.649\ldots = 3.298\ldots$

Hence the width of the oil surface = 3.3 m (2 sig figs)

(b) What other depth of oil would give the same surface width. **1 RE**

By symmetry, rotate the diagram through 180°,

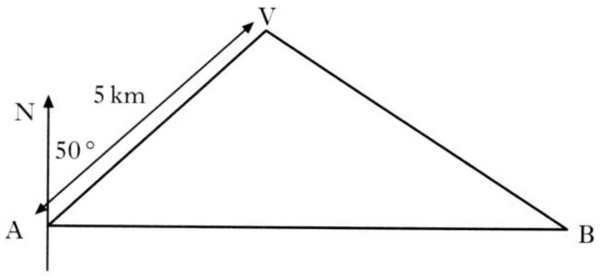
Diameter of tank is 4.2 metres.

And the other depth of oil = 4.2 – 3.4 = 0.8 metres.

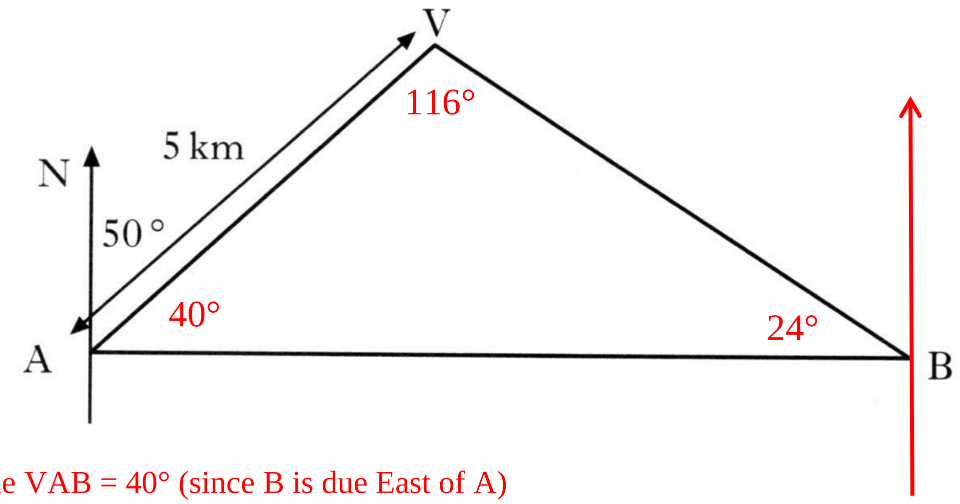
6. David walks on a bearing of 050° from hostel A to a viewpoint V, 5 kilometres away.

Hostel B is due east of hostel A.

Susie walks on a bearing of 294° from hostel B to the same viewpoint.



Calculate the length of AB, the distance between the two hostels. **5 KU**



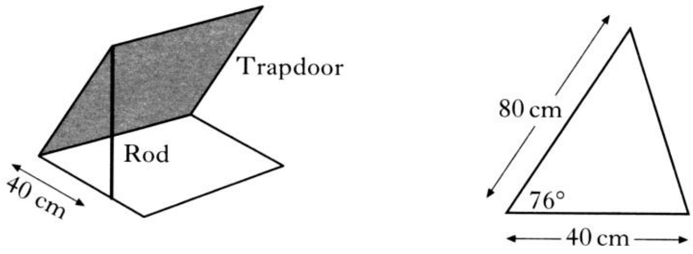
Angle VAB = 40° (since B is due East of A)
Angle VBA = 24° (since bearing of A from B corresponds to 270°, so 294 – 270 = 24°)
Angle AVB = 116° (angles in a triangle add up to 180°)

Configuration of triangle is ASA, so use sine rule.

$$\frac{AB}{\sin 116°} = \frac{5}{\sin 24°} \rightarrow AB = \frac{5\sin 116°}{\sin 24°} = 11.0488....$$

Hence distance between two hostels is 11.0 km (3 sig. fig)

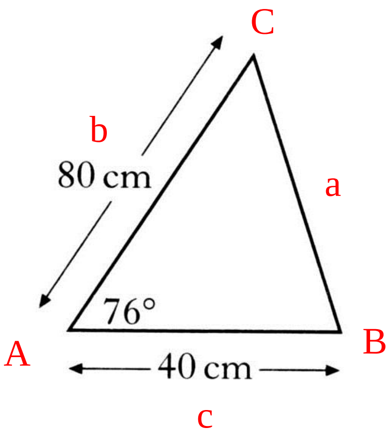
7. A square trapdoor of side 80 centimetres is held open by a rod as shown.



The rod is attached to the corner of the trapdoor and placed 40 centimetres along the edge of the opening.

The angle between the trapdoor and the opening is 76°.

Calculate the length of the rod to 2 significant figures. **4 KU**



Label triangle.

Configuration of triangle is SAS, so use cosine rule.

$a^2 = b^2 + c^2 - 2bcCosA$

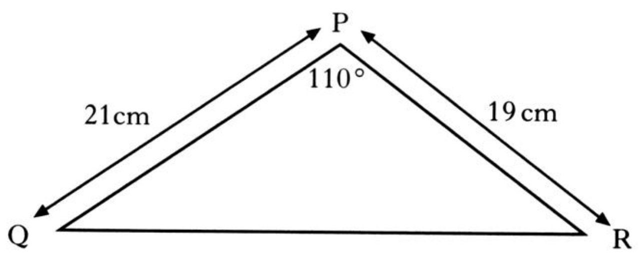
$a^2 = 80^2 + 40^2 - 2 \times 80 \times 40 \times Cos\,76°$

$a^2 = 6451.699...$

$a = 80.322....$

Length of rod = 80 cm (2 sig fig)

8.



Calculate the area of triangle PQR. **4 KU**

Area of triangle = ½ a b sin C

= ½ × 19 × 21 × sin 110° = 187.468…… = 188 cm^2 (3 sig fig)

---

9. Solve algebraically the equation

$$4\sin x° + 1 = -2 \qquad 0 \leq x < 360$$

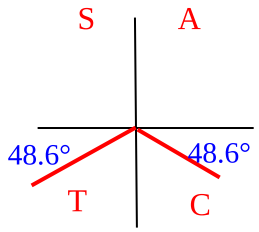
3 KU

$$4\sin x = -3$$

$$\sin x = -\frac{3}{4}$$ **(NB: ignore negative at this stage)**

$$acute\ x = \sin^{-1}\left(\frac{3}{4}\right) = 48.6°$$

Use ASTC sine is negative, so quadrants 3 & 4.



Hence: $x = 180 + 48.6 = 228.6°$

Or $x = 360 - 48.6 = 311.4°$

---

10. (a) Factorise

$$4x^2 - y^2$$

1 KU

Difference of two squares: $(2x+y)(2x-y)$

(b) Hence simplify

$$\frac{4x^2 - y^2}{6x+3y}$$

2 KU

Note denominator has common factor of 3, and numerator is part a)

So: $\frac{4x^2 - y^2}{6x+3y} \rightarrow \frac{(2x+y)^1(2x-y)}{3(2x+y)^1} \rightarrow \frac{2x-y}{3}$

---

11. Factorise fully

$$5x^2 - 45$$

2 KU

Note there is a common factor of 5: $5(x^2 - 9)$

Now difference of two squares: $5(x+3)(x-3)$

---

12. Express as a single fraction in its simplest form

$$\frac{1}{p}+\frac{2}{p+5}$$

2 KU

Common denominator is $p(p+5)$

$$\frac{1}{p}+\frac{2}{p+5} \rightarrow \frac{(p+5)}{p(p+5)}+\frac{2p}{p(p+5)} \rightarrow \frac{p+5+2p}{p(p+5)} \rightarrow \frac{3p+5}{p(p+5)}$$

---

13. Solve the equation

$$\frac{2}{x}+1=6$$

3 KU

Get rid of the fraction, multiply throughout by x.

$$\frac{2}{x}+1=6 \rightarrow 2+x=6x \rightarrow 2=5x \rightarrow x=\frac{2}{5}$$

---

14. Given $f(x)=4\sqrt{x}+\sqrt{2}$

(a) Find the value of $f(72)$ as a surd in its simplest form. **3 KU**

$$f(x)=4\sqrt{x}+\sqrt{2}$$
$$f(72)=4\sqrt{72}+\sqrt{2}$$
$$f(72)=4\sqrt{36\times 2}+\sqrt{2}$$
$$f(72)=4\times 6\sqrt{2}+\sqrt{2}$$
$$f(72)=24\sqrt{2}+\sqrt{2} = 25\sqrt{2}$$

(b) Find the value of t, given that $f(t)=3\sqrt{2}$. **3 RE**

$$f(x)=4\sqrt{x}+\sqrt{2}$$
$$f(t)=4\sqrt{t}+\sqrt{2}$$
$$3\sqrt{2} = 4\sqrt{t}+\sqrt{2}$$
$$2\sqrt{2} = 4\sqrt{t}$$
$$\sqrt{2} = 2\sqrt{t}$$

square each side

$$2=4t \rightarrow t=\frac{1}{2}$$

---

15. Given $P = \frac{2(m-4)}{3}$

Change the subject of the formula to m. **3 KU**

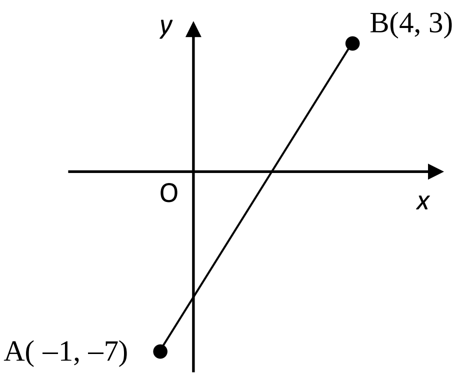
Multiply throughout by 3: $3P = 2(m-4)$

Break bracket: $3P = 2m - 8$

Re-arrange: $3P + 8 = 2m \rightarrow m = \frac{3P+8}{2}$

---

16. In the diagram, A is the point (–1, 7) and B is the point (4, 3).



(a) Find the gradient of the line AB. **1 KU**

Gradient = $m = \frac{rise}{run} \left(or \ \frac{y_2 - y_1}{x_2 - x_1} \right) = \frac{3-(-7)}{4-(-1)} = \frac{10}{5} = 2$

(b) AB cuts the *y*-axis at the point (0, –5).
Write down the equation of the line AB **1 KU**

The y-intercept is c = -5 and since m = 2 then using y = mx + c

We get: $y = 2x - 5$

(c) The point (3*k*, *k*) lies on AB
Find the value of k. **2 RE**

Since point (3*k*, *k*) lies on the line, then it satisfies the equation of the line.

Thus: $k = 2(3k) - 5$

Simplifying: $k = 6k - 5 \rightarrow 5 = 5k \rightarrow k = 1$

---

17. Aaron saves 50 pence and 20 pence coins in his piggy bank.

Let x be the number of 50 pence coins in his bank.

Let y be the number of 20 pence coins in his bank.

(a) There are 60 coins in his bank.
Write down an equation in x and y to illustrate this information. **1 KU**

$$x + y = 60$$

(b) The total value of the coins is £17.40.
Write down another equation in x and y to illustrate this information. **1 KU**

$$50x + 20y = 1740$$

(c) Hence find algebraically the number of 50 pence coins Aaron has in his piggy bank. **3 RE**

We need to find x, so eliminate y.

$$x + y = 60 \qquad (1)$$
$$50x + 20y = 1740 \qquad (2)$$

Multiply (1) by 20

$$20x + 20y = 1200 \qquad (3)$$
$$50x + 20y = 1740 \qquad (4)$$

Subtract (4) – (3)

$$30x = 540$$
$$x = 18$$

He has 18 fifty pence coins in his piggy bank.

---

18. Solve the equation

$$2x^2 + 3x - 7 = 0$$

Give your answers correct to 1 decimal place. **4 KU**

Use the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Where a = 2, b = 3, c = -7

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-7)}}{2(2)} \rightarrow x = \frac{-3 \pm \sqrt{9 + 56}}{4} \rightarrow x = \frac{-3 \pm \sqrt{65}}{4}$$

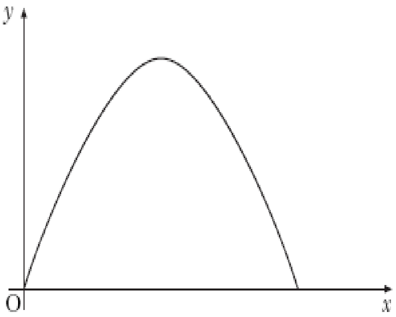
So $\rightarrow x = \dfrac{-3 + \sqrt{65}}{4}$ *or* $x = \dfrac{-3 - \sqrt{65}}{4}$ i.e. $x = 1.265...$ *or* $x = -2.765....$

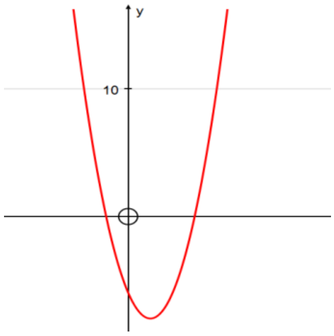
$x = 1.3$ *or* $x = -2.8$ $(1\,dec.\ place)$

19. The diagram shows part of the graph of a quadratic function, with equation of the form

$$y = k(x-a)(x-b)$$

The graph cuts the y-axis at (0, –6) and the x-axis at (–1, 0) and (3, 0)





Note: It appears that the incorrect diagram was shown with this question. The correct diagram is shown above on the right.

(a) Write down the values of a and b. **2 KU**

a and b are where it cuts the x-axis: so a = -1 and b = 3

(b) Calculate the value of k. **2 KU**

Putting in values of a and b. $y = k(x+1)(x-3)$

The graph cuts the y-axis when x = 0 and y = -6, so put this point in the equation.

$$-6 = k(0+1)(0-3) \rightarrow -6 = k(1)(-3)$$
$$-6 = -3k \rightarrow k = 2$$

(c) Find the coordinates of the minimum turning point of the function **2 RE**

Turning point is mid-way between the roots of x = -1 and x = 3

so x coordinate of turning point is 1

To find the y-coordinate of turning point, put x = 1 into the equation

$$y = 2(x+1)(x-3) \rightarrow y = 2(1+1)(1-3) \rightarrow y = 2(2)(-2) \rightarrow y = -8$$

Coordinates of minimum turning point are : (1, -8)

---

20. Given that $x^2 - 10x + 18 = (x-a)^2 + b$

Find the values of a and b. **3 KU**

If these two expressions are the same, then the coefficients of x^2, x and the constant must be the same.

Expand the right hand side: $(x-a)^2 + b \rightarrow x^2 - 2ax + a^2 + b$

Now compare this with: $x^2 - 10x + 18$

And we can see that: $-2a = -10 \rightarrow a = 5$

And using the value for a: $a^2 + b = 18 \rightarrow 25 + b = 18 \rightarrow b = -7$

---

21. (a) Simplify $2\sqrt{75}$ **2 KU**

$2\sqrt{75} \rightarrow 2\sqrt{25\times3} \rightarrow 10\sqrt{3}$

(b) Evaluate $2^0 + 3^{-1}$ **2 KU**

$2^0 + 3^{-1} \rightarrow 1 + \frac{1}{3} \rightarrow 1\frac{1}{3}$

---

22. (a) Simplify $2a \times a^{-4}$ **1 KU**

Adding the indices: $2a \times a^{-4} \rightarrow 2a^{-3}$

(b) Solve for x. $\sqrt{x} + \sqrt{18} = 4\sqrt{2}$

We need to get √18 in terms of √2.

$$\sqrt{x} + \sqrt{18} = 4\sqrt{2}$$
$$\sqrt{x} + \sqrt{9\times2} = 4\sqrt{2}$$
$$\sqrt{x} + 3\sqrt{2} = 4\sqrt{2}$$
$$\sqrt{x} = \sqrt{2}$$
$$x = 2$$

---

23. Solve the equation $3x+1 = \frac{x-5}{2}$ **3 KU**

Multiply throughout by 2 to remove the fraction:

$$3x+1 = \frac{x-5}{2}$$
$$6x + 2 = x - 5$$
$$5x = -7$$
$$x = -\frac{7}{5} = -1\frac{2}{5}$$

---

24. Factorise fully $2m^2 - 18$ **2 KU**

Take out 2 as a common factor: $2(m^2 - 9)$

Now it is difference of two squares: $2(m+3)(m-3)$

---

25. Given that $f(x) = 5 - x^2$ evaluate $f(-3)$ **2 KU**

$$f(-3) = 5 - (-3)^2$$
$$f(-3) = 5 - 9 \rightarrow f(-3) = -4$$

---

26. Olga normally runs a total distance of 28 miles per week.

She decides to increase her distance by 10% a week for the next four weeks.

How many miles will she run in the fourth week ? **3 KU**

An increase of 10% a multiplier of 100% + 10% i.e. 110% or $\times 1.1$

So in 4th week she will run $28 \times 1.1^4 = 40.99.... = 41\ miles\ (2\ sig\ figs)$

---

27. A car is valued at £3780.

This is 16% less than last year's value.

What was the value of the car last year ? **3 KU**

It is now worth 100% - 16% = 84% of its original value.

84% = £3780

1% = £3780 ÷ 84

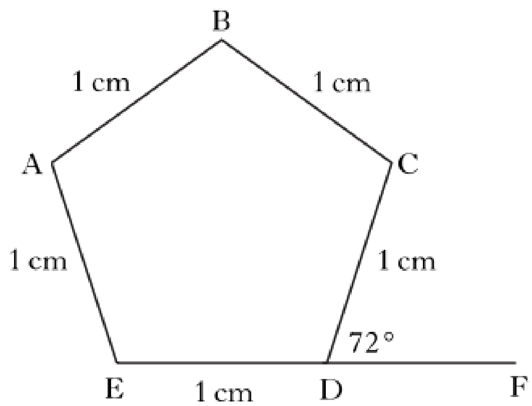
100% = £3780 ÷ 84 × 100 = £4500.

---

28. ABCDE is a regular pentagon with each side 1 cm.

Angle CDF is 72°.

EDF is a straight line.



(a) Write down the size of angle ABC **1 KU**

Angle CDE = 180 – 72° = 108°. All internal angles are the same

So, angle ABC = 108°

(b) Calculate the length of AC **3 KU**

ABC is a triangle. Configuration is SAS so use cosine rule

(You could also use SOH-CAH-TOA by finding angle BAC and drawing a perpendicular down from B as ABC is isosceles)

$$b^2 = c^2 + a^2 - 2ca\,CosB$$

$$b^2 = 1^2 + 1^2 - 2 \times 1 \times 1 \times Cos108°$$

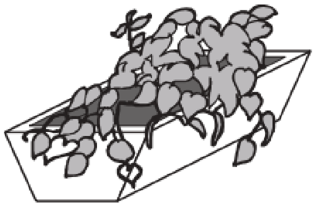
$$b^2 = 2.618..... \quad b = 1.618.....$$

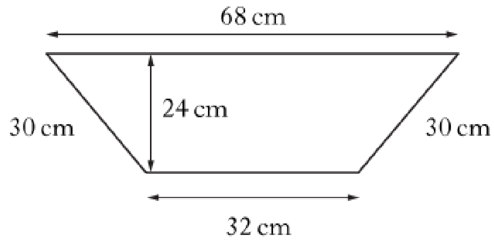
Length of AC = 1.6 cm (2 sig fig)

---

29. A flower planter is in the shape of a prism.

The cross section is a trapezium with dimensions shown.



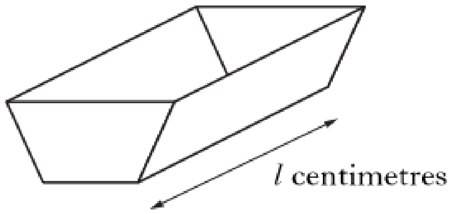


(a) Calculate the area of the cross-section of the planter. **2 KU**

Area of trapezium = $\frac{1}{2}(a+b)h \rightarrow \frac{1}{2}(68+32)\times 24 \rightarrow 1200\ cm^2$

You could also split the shape into two triangles and a rectangle

(b) The volume of the planter is 156 litres.



Calculate the length l centimetres, of the planter. **3 RE**

Volume of a prism = Al Change 156 litres to cm^3 (× 1000) = 156,000 cm^3

$156000 = 1200 \times l$ $l = \frac{156000}{1200} = 130\ cm$

---

30. Tom and Samia are paid the same hourly rate.

Harry is paid $\frac{1}{3}$ more per hour than Tom.

Tom worked 15 hours, Samia worked 8 hours and Harry worked 12 hours.

They were paid a total of £429.

How much was Tom paid ? **3 KU**

Let Tom and Samia each be paid £x per hour.

Harry is paid $\frac{1}{3}$ per hour more than Tom,

so Harry is paid $\left(1+\frac{1}{3}\right)x \rightarrow \left(\frac{3}{3}+\frac{1}{3}\right)x \rightarrow \frac{4}{3}x$ £ / hr.

Thus: $15x+8x+12\left(\frac{4}{3}x\right)=429 \rightarrow 23x+16x=429 \rightarrow 39x=429 \rightarrow 11$

Tom is paid £11 per hour.

---

31. Evaluate 40% of £11.50 - £1.81 **2 KU**

Work out percentage first: 10% = £1.15, so 40% = £4.60

Now do subtraction: £4.60 - £1.81 = £2.79

---

32. Evaluate $\frac{2}{5} \div 1\frac{1}{10}$ **2 KU**

Change to improper fraction: $\frac{2}{5} \div 1\frac{1}{10} \rightarrow \frac{2}{5} \div \frac{11}{10} \rightarrow \frac{2}{\cancel{5}^{1}} \times \frac{\cancel{10}^{2}}{11} \rightarrow \frac{4}{11}$

---

33. Change the subject of the formula to s.

$$t = \frac{7s+4}{2}$$

3 KU

Multiply both sides by 2: $2t = 7s + 4 \rightarrow 2t - 4 = 7s \rightarrow s = \frac{2t-4}{7}$

---

34. A bag contains 27 marbles. Some are black and some are white.

The probability that a marble chosen at random is black is $\frac{4}{9}$

(a) What is the probability that a marble chosen at random is white ? **1 KU**

P(White) = $\frac{5}{9}$

(b) How many white marbles are in the bag ? **1 RE**

Number of white marbles = $\frac{5}{9} \times 27 = 15$

---

35. Cleano washing powder is on special offer.

Each box on special offer

contains 20% more powder than the standard box.

A box on special offer contains 900 grams of powder.

How many grams of powder does the standard box contain ? **3 KU**

Special offer is 20% more which is 120% of standard box.

So: 120% = 900

1% = 900 ÷ 120

100% = 900 ÷ 120 × 100

$\frac{900}{120} \times \frac{100}{1} \rightarrow \frac{90}{12} \times \frac{100}{1} \rightarrow \frac{\cancel{90}^{30}}{\cancel{12}^{4}} \times \frac{100}{1} \rightarrow \frac{3000}{4} = 750g$

---

36. (a) Simplify $\sqrt{2} \times \sqrt{18}$ **1 KU**

$\sqrt{2} \times \sqrt{18} \rightarrow \sqrt{36} \rightarrow 6$

(b) Simplify $\sqrt{2} + \sqrt{18}$ **1 KU**

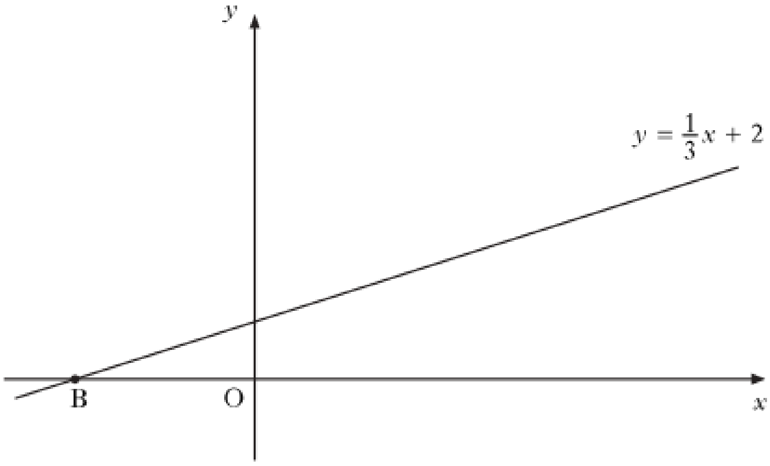
$\sqrt{2} + \sqrt{18} \rightarrow \sqrt{2} + \sqrt{9 \times 2} \rightarrow \sqrt{2} + 3\sqrt{2} \rightarrow 4\sqrt{2}$

(c) Hence show that $\dfrac{\sqrt{2} \times \sqrt{18}}{\sqrt{2} + \sqrt{18}} = \dfrac{3\sqrt{2}}{4}$ **2 KU**

$\dfrac{\sqrt{2} \times \sqrt{18}}{\sqrt{2} + \sqrt{18}} \rightarrow \dfrac{6}{4\sqrt{2}} \rightarrow \dfrac{6\sqrt{2}}{4\sqrt{2}\sqrt{2}} \rightarrow \dfrac{6\sqrt{2}}{8} \rightarrow \dfrac{3\sqrt{2}}{4}$

---

37. Part of the graph of the straight line with equation $y = \frac{1}{3}x + 2$ is shown below.



(a) Find the coordinates of point B. **2 KU**

At B, y = 0 and so: $y = \frac{1}{3}x + 2 \rightarrow 0 = \frac{1}{3}x + 2 \rightarrow 0 = x + 6 \rightarrow x = -6$

Hence: *B* is *B*(-6, 0)

(b) For what values of *x* is *y* < 0 **1 RE**

$y < 0$ when the line is below the *x*-axis i.e. when $x <$ -6

---

38. It is estimated that an iceberg weights 84 000 tonnes.

As the iceberg moves into warmer water, its weight decreases by 25% each day.

What will the iceberg weigh after 3 days in the warmer water ?

Give your answer **correct to three significant figures.** **4 KU**

Decrease of 25% = multiplier of 100% – 25% = 75% or 0.75

After 3 days iceberg will weigh: $84000 \times 0.75^3 = 35437.5 = 35400 \textit{ tonnes } (3 \, s.f.)$

39. Expand and fully simplify $x(x-1)^2$ **2 KU**

$x(x-1)^2 \rightarrow x(x-1)(x-1) \rightarrow x(x^2-2x+1) \rightarrow x^3-2x^2+x$

---

40. A machine is used to put drawing pins into boxes.

A sample of 8 boxes is taken and the number of drawing pins in each is counted.

The results are: 102 102 101 98 99 101 103 102

(a) Calculate the mean and standard deviation of this sample **3 KU**

Mean = $\frac{808}{8} = 101$

Draw and complete table

Use formula:

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

$$s = \sqrt{\frac{20}{7}} = 1.6903..... = 1.7$$

x	$x-\bar{x}$	$(x-\bar{x})^2$
102	1	1
102	1	1
101	0	0
98	-3	9
99	-2	4
101	0	0
103	2	4
102	1	1
808		20

Mean = 101 pins with a standard deviation of 1.7 pins.

(b) A sample of 8 boxes is taken from another machine.

This sample has a mean of 103 and a standard deviation of 2.1

Write down two valid comparisons between the samples. **2 RE**

The mean number of pins in the first machine sampled has a lower mean (101 pins)
So there are less pins in the box on average.

However, the standard deviation of the first machine is lower (1.7 pins),
indicating a more consistent number of pins in a box.

---

41. Use the quadratic formula to solve the equation:

$$3x^2-5x+7=0$$

Give your answers correct to 1 decimal place. **4 KU**

Note: Misprint in this question: should be $3x^2-5x-7=0$

Use the quadratic formula: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ Where a = 3, b = -5, c = -7

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(3)(-7)}}{2(3)} \rightarrow x=\frac{5\pm\sqrt{25+84}}{6} \rightarrow x=\frac{5\pm\sqrt{109}}{6}$$

So $\rightarrow x=\frac{5+\sqrt{109}}{6}$ *or* $x=\frac{5-\sqrt{109}}{6}$ i.e. $x=2.573...$ *or* $x=-0.9067....$

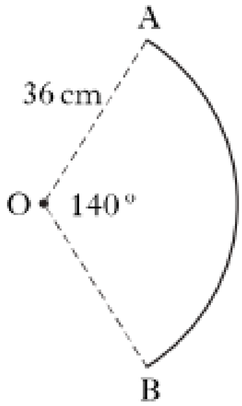
$x=2.6$ *or* $x=-0.9$ (1*dec. place*)

---

42. A circle, centre O, has radius 36 cm.

Part of this circle is shown.

Angle AOB = 140°

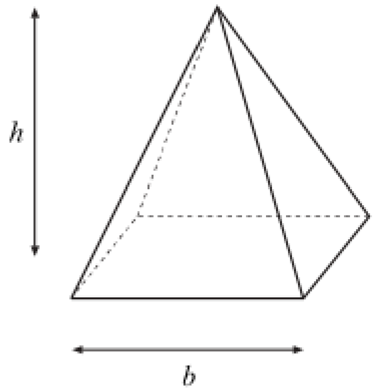


Calculate the length of arc AB **3 KU**

Length of arc = $\frac{140}{360} \times \pi \times 2 \times 36 = 87.964.... = 88.0\,cm\ (3\,sig.\,fig.)$

---

43. The height, h , of a square based pyramid varies directly as its volume V and inversely as the square of the length of the base, b.



(a) Write down an equation connecting h, V and b. **2 KU**

$$h \propto V \quad h \propto \frac{1}{b^2} \quad \rightarrow \quad h = \frac{kV}{b^2}$$

A square-based pyramid of height 12 cms has a volume of 256 cm^3 and length of base 8 cm.

(b) Calculate the height of a square-based pyramid of volume 600 cm^3 and length of base 10 cm. **3 KU**

First find k using information above: $12 = \frac{k \times 256}{8^2} \rightarrow k = \frac{12 \times 64}{256} \rightarrow k = 3$

So, $h = \frac{3V}{b^2}$ Now use the formula: $h = \frac{3 \times 600}{10^2} \rightarrow h = \frac{3 \times 600}{100} \rightarrow h = 18\ cm$

---

44. The depth of water, D metres, in a harbour is given by the formula

$$D = 3 + 1.75 \sin 30h°$$

Where h is the number of hours after midnight.

(a) Calculate the depth of water at 5 am. **2 KU**

Put $h = 5$ in the formula: $D = 3 + 1.75 \sin 30 \times 5°$

Now evaluate: $D = 3 + 1.75 \sin 150° \rightarrow D = 3 + 0.875 = 3.875$ *metres.*

(b) Calculate the maximum difference in depth of the water in the harbour.
Do not use a trial and improvement method. **2 RE**

The only value that can vary is h, so look at the max and min of sine function.
Max value of sine is 1, so Max D = 3 + 1.75 = 4.75 metres.
Min value of sine is -1, so Min D = 3 – 1.75 = 1.25 metres.

Maximum difference in depth is 4.75 – 1.25 = 3.5 metres.

---

45. Evaluate $4\frac{1}{3} - 1\frac{1}{2}$ **2 KU**

Deal with whole numbers. Then use common denominator of 6:

$$4\frac{1}{3} - 1\frac{1}{2} \rightarrow 3 + \frac{1}{3} - \frac{1}{2} \rightarrow 3 + \frac{2}{6} - \frac{3}{6} \rightarrow 3 - \frac{1}{6} \rightarrow 2\frac{5}{6}$$

---

46. (a) Factorise $x^2 - 4y^2$ **1 KU**

Difference of two squares: $x^2 - 4y^2 \rightarrow (x+2y)(x-2y)$

(b) Expand and simplify $(2x-1)(x+4)$ **1 KU**

Use FOIL: $(2x-1)(x+4) \rightarrow 2x^2 + 8x - x - 4 \rightarrow 2x^2 + 7x - 4$

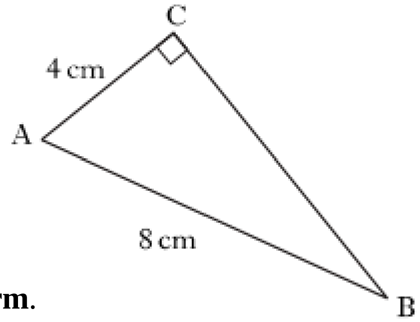
(c) Expand $x^{\frac{1}{2}}(3x + x^{-2})$ **2 KU**

$x^{\frac{1}{2}}(3x + x^{-2}) \rightarrow 3x.x^{\frac{1}{2}} + x^{\frac{1}{2}}.x^{-2}$ Now add indices: $\rightarrow 3x^{\frac{3}{2}} + x^{-\frac{3}{2}}$

---

47. In triangle ABC:

- Angle ACB = 90°
- AB = 8 cm
- AC = 4 cm



Calculate the length of BC.

Give your answer **as a surd in its simplest form**. **3 KU**

Use Pythagoras: $8^2 = 4^2 + BC^2 \quad \rightarrow \quad BC^2 = 8^2 - 4^2 \quad \rightarrow \quad BC^2 = 64 - 16 = 48$

$BC = \sqrt{48} \quad = \quad \sqrt{16 \times 3} \quad = \quad 4\sqrt{3}$

---

48. There are 4 girls and 14 boys in a class.

A child is chosen at random and is asked to roll a die, numbered 1 to 6.

Which of these is more likely ?

A: the child is female

OR

B: the child rolls a 5

Justify your answer **3 RE**

P(child is female) = $\frac{4}{18}$ P(Child rolls a 5) $= \frac{1}{6} = \frac{3}{18}$

Hence more likely that child is female since $\frac{4}{18} > \frac{3}{18}$

---

49. A formula used to calculate the flow in a pipe is

$$f = \frac{kd^2}{20}$$

Change the subject of the formula to d. **3 KU**

Multiply both sides by 20: $\rightarrow \quad 20f = kd^2 \quad \rightarrow \quad d^2 = \frac{20f}{k} \quad \rightarrow \quad d = \sqrt{\frac{20f}{k}}$

---

50. One atom of gold weight 3.27×10^{-22} grams.

How many atoms will there be in 1 kg of gold ?

Give your answer **in scientific notation correct to 2 significant figures.** **3 KU**

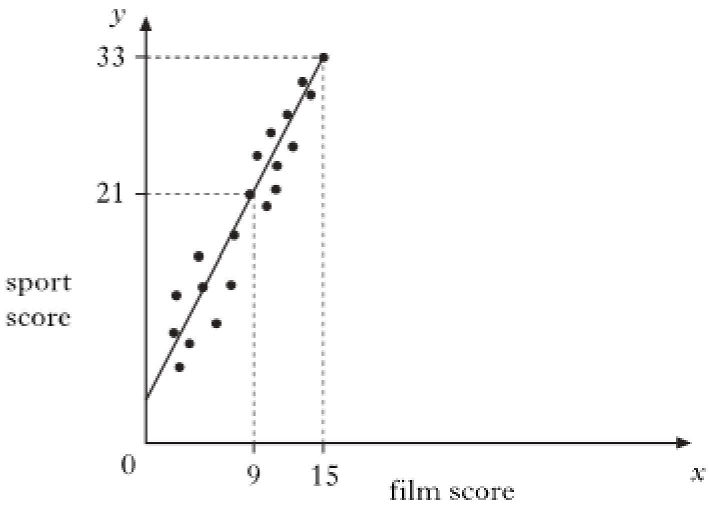
Consider 1 apple weighs 100g – how many apples in 1 kg: 1000 ÷ 100 = 10

Hence number of atoms in 1kg gold is: $1000 \div \left(3.27 \times 10^{-22}\right) = 3.058..... \times 10^{24}$

$= 3.1 \times 10^{24}$ (2 *sig. fig.*)

---

51. Teams in a quiz answerr questions on film and sport.
This scatter graph shows the scores of some of the teams.



A line of best fit is drawn as shown above.

(a) Find the equation of this straight line. **4 KU**

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{33-21}{15-9} = \frac{12}{6} = 2$ Put this into $y = mx + c$

Hence: $y = 2x + c$ and use one of the points to find c (say 9, 21)

Thus: $21 = 2(9) + c$ so $c = 21 - 18 = 3$

Equation is: $y = 2x + 3$

(b) Use this equation to estimate the sport score for a team
With a film score of 20 **2 RE**

Put $x = 20$ into the equation: $y = 2(20) + 3$ $y = 43$. So estimate of sport score = 43

---

52. (a) The air temperature, t° Celsius, varies inversely as the square of
The distance, d metres, from a furnace.

Write down a formula connecting t and d. **2 KU**

$$t \propto \frac{1}{d^2} \rightarrow t = \frac{k}{d^2}$$

(b) At a distance of 2 metres from the furnace, the air temperature is 50 °C.

Calculate the air temperature at a distance of 5 metres from the furnace. **3 KU**

First find k: $50 = \frac{k}{2^2} \rightarrow k = 200$ so equation is: $t = \frac{200}{d^2}$

Now use equation: $t = \frac{200}{5^2} \rightarrow t = \frac{200}{25} \rightarrow t = 8\ °C$
